MATHEMATICS

Mob. : 9470844028
9546359990

AIM POINTS
MATHEMATICS
DIR. FIROZ AHMAD

Ram Rajya More, Siwan

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THE PLANE

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. The general eqation of first degree in x, y, z i.e., $ax + by + cz + d - 0$ always represents a plane.
2. In the equation $ax + by + cz + d = 0$, the direction ratios of normal to the plane are proportional to a, b, c.
3. A vector normal to the plane $ax + by + cz + d = 0$ is $\vec{n} = a\hat{i} + b\hat{j} + c\hat{k}$.
4. If I, m, n are the direction cosines of normal to a plane which is at a distance p from the ogigin, then the cartesian equation of the plane is

   $ix + my + nz = p.$

   This is known as the normal form of a plane.
5. The vector equation of a plane passing through a point having postition vector $\vec{a}$ and normal to $\vec{n}$ is

   $(\vec{r} \cdot \vec{a}) \cdot \vec{n} = 0$ or, $\vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$
6. The cartesian equation of a plane passing through (x_1, y_1, z_1) and having direction ratios proportional to a, b, c for its normal is

   $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
7. The vector equation of a plane having $\vec{n}$ as a unit vector normal to it and at a distance 'd' from the origin is $\vec{r} \cdot \vec{n} = d$.

   If l, m, n are direction cosines of the normal to the plane, then its vector equations

   $\vec{r}(l\hat{i} + m\hat{j} + n\hat{k}) = d$

   This is the vector equation of the normal form of a plane.
8. The vector equation of a plane passing through points having position vectors $\vec{a} \cdot \vec{b}$ and $\vec{c}$ is

   $\vec{r}(\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = \vec{a}(\vec{b} \times \vec{c})$
9. A vector normal to the plane passing through points A ($\vec{a}$), B, ($\vec{b}$) and C ($\vec{c}$) is

   $\overrightarrow{AB} \times \overrightarrow{AC}$ or, $\overrightarrow{BC} \times \overrightarrow{BA}$ or, $\overrightarrow{CB} \times \overrightarrow{CA}$

   i.e., $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} \cdot \vec{c} \times \vec{a}$
10. The cartesian equation of a plane intercepting lengths a, b and c with X, Y and Z axes respectively is

    $$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$
11. The cartesian eqation of a plane passing through points (x_1, y_1, z_1) (x_2, y_2, z_2) and (x_3, y_3, z_3) is

$$\begin{vmatrix} x & y & z & 1 \\ x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \end{vmatrix} = 0 \qquad \text{or,} \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

12. The angle between two palnes is defined as the angle between their normals.

(i) If $\vec{r}.\vec{n}_1 = d$ and $\vec{r}.\vec{n}_2 = d_2$ are two planes inclined at an angle θ, then

$$\cos\theta = \frac{\vec{n}_1.\vec{n}_2}{|\vec{n}_1||\vec{n}_2|}$$

These planes are parallet, if $\vec{n}_1$ is paralled to $\vec{n}_2$.

These planes are perpendicular, if $\vec{n}_1 \cdot \vec{n}_2 = 0$

(ii) if $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are cartesian equations of thwo planes inclined at an angle θ, then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

The planes are parallel, if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

The planes are perpendicular, if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

13. The vector equation of plane passing through a point having position vector $\vec{a}$ and parallel to vectors $\vec{b}$ and $\vec{c}$ is

$\vec{r} = \vec{a} + m\vec{b} + n\vec{c}$, where m and n are parameters.

or, $\vec{r}(\vec{b} \times \vec{c}) = \vec{a}(\vec{b} \times \vec{c})$

14. The vector equation of the plane passing through points having position vectors, $\vec{a}, \vec{b}$ and $\vec{c}$

$\vec{r} = (1 - m - n)\ \vec{a}\ m\vec{b}\ n\vec{c}$

$\vec{r}(\vec{a} \times \vec{b}) \cdot \vec{r}(\vec{b} \times \vec{c}) + \vec{r}(\vec{c} \times \vec{a}) = \vec{a}(\vec{b} \times \vec{c})$

15. The equation of a plane paralled to the plane

(a) $\vec{r} \cdot \vec{n} = d$ is $\vec{r} \cdot \vec{n} = d_1$

(b) $ax + by + cz + d = 0$ is $ax + by + cz + \gamma = 0$

16. The length of a plane paralled to the plane

$ax + by + cz + d = 0$ is

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

and the coordinates (α, β, γ) of the foot of the perpendicular are given by

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = -\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)$$

The coordinates of the image of the poing (x_1, y_1, z_1) in the plane $ax + by + cz + d = 0$ are given by

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = \frac{\gamma - z_1}{c} = -2\left(\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}\right)$$

17. The distance between the paralle plane $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is given by

$$\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$$

18. The equation of the family of planes containing the line
$a_1 x + b_1 y + c_1 z + d_1 = 0 = a_2 x + b_2 y + c_2 z + d_2$ is
$(a_1 x + b_1 y + c_1 z + d_1) + \gamma\ (a_2 x + b_2 y + c_2 z + d_2) = 0$, where γ is a parameter.

19. the equations of the planes bisecting the angles between the planes $a_1 x + b_1 y + c_1 z + d_1 = 0$ and $a_2 x + b_2 y + c_2 z + d_2 = 0$ are given by

$$\frac{ax_1 + by_1 + cz_1 + d_1}{\sqrt{a^2 + b^2 + c^2}} = \pm\frac{ax_2 + by_2 + cz_2 + d_2}{\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

20. The angle θ between aline and a plane $ax + by + cz + d = 0$ is the complement of the angle between the line and normal to the plane and is given by

$$\sin\theta = \frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$$

The angle θ between the line $\vec{r} = \vec{a}\ \gamma \vec{b}$ and the plane $\vec{r} \cdot \vec{n} = d$ is given by

$$\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}$$

A line is parallel to a plane if it is perpendicular to the normal to the plane.
A line is perpendicular to a plane if it is parallel to the normal to the plane.

21. The line $\vec{r} = \vec{a} + \gamma \vec{b}$ line in the plane $\vec{r} \cdot \vec{n} = d$, if

$\vec{r} \cdot \vec{n} = d$ and $\vec{b} \cdot \vec{n} = 0$

The line $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ line inthe plane

$ax + by + cz + d = 0$, if $ax_1 + by_1 + cz_1 + d = 0$ and $al + bm + cn = 0$

22. The equation of plane cataining the line

$\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is

$a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$, where $al + bm + cn = 0$

23. Two lines $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and, $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$ are coplanar if

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

and the equation fo the plane containing them is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0 \text{ or, } \begin{vmatrix} x-x_2 & y-y_2 & z-z_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$$

24. Two lines $\vec{r}=\vec{a}_1+\gamma\,\vec{b}_1$ and $\vec{r}=\vec{a}_2+\mu\,\vec{b}_2$ are coplanar, if

$$\vec{a}_1\cdot(\vec{b}_1\times\vec{b}_2)=\vec{a}_2(\vec{b}_1\times\vec{b}_2)$$

EXERCISE-1

1. If form a point P (a, b, c) perpendiculars PA and PB are drawn to yz and zx-planes, find the equation of the plane OAB.
2. Find the equation of the plane passing through the following points :
   (i) (e, 1, 0), (3, – 2, – 2) and (3, 1, 7)
   (ii) (– 5, 0, – 6), (– 3, 10, – 9) and (– 2, 6, –6)
   (iii) (1, 1, 1), (1, – 1, 2) and (– 2, – 2, 2)
   (iv) (2, 3, 4), (–3, 5, 1) and (4, –1, 2)
   (v) (0, –1, 0) (3, 3, 0) and (1, 1, 1)
3. Show that the following points are coplanar :
   (i) (0, –1, 0), (2, 1, –1), (1, 1, 1) and (3, 3, 0)
   (ii) (0, 4, 3), (–1, –5, –3), (–2, –2, 1) and (1, 1, –1)
4. A plane meets the coordinate axes in A, B, C such that the centroid of triangle ABC is the point (p, q, r), Show that the equation fo the plane is $\frac{x}{p}+\frac{y}{q}+\frac{z}{r}=3$
5. Find the angles at which the normal vector to the plane $4x+8y+z=5$ is inclined to the coordinate axes.
6. Find the cartesian form of equation of a plane whose vector equation is
   (i) $\vec{r}.(12\hat{i}-3\hat{j}+4\hat{k})+5=0$
   (ii) $\vec{r}.(-\hat{i}+\hat{j}+2\hat{k})=9$
7. Find the vector and cartesian equations of a plane passing through the point (1, –1, 1) and normal to the line joining the points (1, 2, 5) and (–1, 3, 1).

8. The foot of the perpendicular drawn from the origin to a plane is (12, –4, 3). Find the equation of the plane.
9. Find the equation of the plane passing through the point (2, 3, 1) having 5, 3, 2 as direction ratios of normal to the plane.
10. If the axes are rectangular and P is the point (2, 3, –1), find the equation of the plane through P at right angles to OP.
11. Find the intercepts made on the coordinate axes by the plane $2x + y - 2z = 3$ and find also the direction cosines of the normal to the plane.
12. Show that the normals to the following pairs of planes are perpendicular to each other :
    (i) $x - y + z - 2 = 0$ and $3x + 2y - z + 4 = 0$
    (ii) $\vec{r}.(2\hat{i} - \hat{j} + 3\hat{k})$ and $\vec{r}.(2\hat{i} - 2\hat{j} - 2\hat{k}) = 0$
13. Find the equation of the plane which bisects the line joining the points (–1, 2, 3) and (3, –5, 6) at right angles.
14. Reduce the equation $ex - 3y - 6z = 14$ to the normal form and hence the lenght of perpendicular from the origin to the plane. Also, find the direction cosines of the normal to the plane.
15. Reduce the equation $\vec{r}.(\hat{i} - 2\hat{j} + 2\hat{k}) + 6 = 0$ to normal form and hence find the lenght of perpendicular form the origin to the plane.
16. Write the normal form of the equation of the plane $2x - 3y + 6z + 14 = 0$
17. The direction ratios of the perpendicular from the origin to a plane are 12, –3, 4 and the lenght of the perpendicular is 5. Find the equation of the plane.
18. Find a unit normal vector to the plane $x + 2y + 3z - 6 = 0$.
19. find the equation of a plane which is at a distance of $3\sqrt{3}$ unit from the origin and the normal to which is equally inclined with the coordianate axes.
20. Find the equation of the plane passing thorugh the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and)2, 3, 5). Find also the perpendicular distance of the origin from this plane.
21. Find the angle between the plane :
    (i) $\vec{r}.(2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$ and $\vec{r}.(-\hat{i} + \hat{j}) = 4$
    (ii) $\vec{r}.(2\hat{i} - \hat{j} + 2\hat{k}) = 6$ and $\vec{r}.(3\hat{i} + 6\hat{j} - 2\hat{k}) = 9$
    (ii) $\vec{r}.(2\hat{i} + 3\hat{j} - 6\hat{k}) = 5$ and $\vec{r}.(\hat{i} - 2\hat{j} + 2\hat{k}) = 9$
22. Find the angle between the plane :
    (i) $2x - y + z = 4$ and $x + y + 2z = 3$
    (ii) $x + y - 2z = 3$ and $2x - 2y + z = 5$
    (iii) $x - y + z = 5$ and $x + 2y + z = 9$
    (iv) $2x - 3y + 4z = 1$ and $-x + y = 4$
23. Determine the value of λ for which following o;anes are perpendicular to each other.
    (i) $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) = 7$ and $\vec{r}.(\lambda\hat{i} + 2\hat{j} - 7\hat{k}) = 26$

(ii) $ex - 4y + 3z = 5$ and $x + 2y + \lambda z = 5$

(iii) $3x - 6y - 2z = 7$ and $2x + y - \lambda z = 5$

24. Find the equation of aplane passing through the point (–1, –1, 2) and perpendicular to the planes $3x + 2y - 3z = 1$ and $5x - 4y + z = 5$.
25. Obtain the equation of the plane passing through the point (1, –3, –2) and perpendicular to the planes $x + 2y + 2z = 5$ and $3x + 3y + 2z = 8$.
26. Find the equation of the plane passing thorugh the points (1, –1, 2) and)2, –2, 2) and which is perpendicular to the plane $6x - 2y + 2z = 9$.
27. Find th equation of the plane passing through the points whose coordinates are (–1, 1, 1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.
28. Find the vector equation of the following planes in scalar product form ($\vec{r}\cdot\vec{n} = d$).

(i) $\vec{r} = (2\hat{i} - \hat{k}) + \lambda\hat{i} + \mu(\hat{i} - 2\hat{j} - \hat{k})$

(ii) $\vec{r} = (1 + s - t)\hat{i} + (2 - s)\hat{j} + (2 - 2s + 2t)\hat{k}$

(iii) $\vec{r} = (2\hat{i} + \hat{j}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

(iv) $\vec{r} = \hat{i} - \hat{j} + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(4\hat{i} - 2\hat{j} + 3\hat{k})$

29. Find the cartesian form of the equation of the following planes :

(i) $\vec{r} = (\hat{i} + \hat{j}) + s(-\hat{i} + \hat{j} + 2\hat{k}) + t(\hat{i} + 2\hat{j} + \hat{k})$

(ii) $\vec{r} = (1 + s + t)\hat{i} + (2 - s + t)\hat{j} + (3 - 2s + 2t)\hat{k}$

30. Find the equation of a plane thorugh the intersection of the plane $\vec{r}.(\hat{i} + 3\hat{j} - 2\hat{k}) = 5$ and $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) = 3$ and passing through the point (2, 1, ..2)
31. Find the equation of the plane containing the line of intersection of the plane $x + y + z - 6 = 0$ and $2x + 3y + 4z + 5 = 0$ and passing thorugh the point (1, 1, 1).
32. Find the direction ratios of the normal to the plane passing through the point (2, 1, 3) and the line of intersection of the planes $x + 2y + z = 3$ and $2x - y - z = 5$.
33. Find the equation of the plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$.
34. Find the equation of the plane through the line of intersection of $\vec{r}.(2\hat{i} - 3\hat{j} + 4\hat{k}) = 1$ and perpendicutar to $\vec{r}.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$.
35. Find the cartesion as well as vector equations of the planes thoruhg the intersection of tite planes $\vec{r}.(6\hat{i} + 6\hat{j}) + 12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$ which are at a unit distance form the origin.
36. Find the equation of the plane through the line of of intersection of the planes $x + 2y + 3z + 4 = 0$ and $2x + y - z + 5 = 0$ and which is perpendicular to the plane $5x + 3y - 6z + 8 = 0$.
37. Find the equation of the plane thoruhg the line of intersection of the planes $x + 2y + 3z + 4 = 0$ and $x - y + z + 3 = 0$ and passing through the origin.

38. find the equation of the plane thorugh th line of intersection of planes $\vec{r}.(\hat{i} + 3\hat{j}) + 6 = 0$ and $\vec{r}.(3\hat{i} - \hat{j} - 4\hat{k}) = 0$, which is a unit distance from the origin.

39. Find the equation of the plane passing through the intersection of the planes $2x + 3y - z + 1 = 0$ and $x + y - 2z + 3 = 0$ and perpendicular to the plane $3x - y - 2z - 4 = 0$.

40. Find the equation of plane that contains the line of intersection of the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$ and $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane $\vec{r}.(5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0$.

41. Find equation fo the plane passing throuth intersection of the planes $\vec{r}.(2\hat{i} + \hat{j} + 3\hat{k}) = 7$ and $\vec{r}.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 9$ and the point (2, 1, 3).

42. Find equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ which is at a unit distance from the point (1, 2, 3).

43. If a variable plane at a contstant distance p from the origin meets the coordinate axes in points A. B and C respectively. Thorugh these points, planes are drawn parallel to the coordinate planes. Show that the locus of the point of intersection is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$$

44. Find the distance between the point P(6, 5, 9) and the plane determined by the points A(3, –1, 2), B (5, 2, 4) nad C (–1, –1, 6).

45. Find the distance of point $2\hat{i} - \hat{j} - 4\hat{k}$ from the plane $\vec{r}.(3\hat{i} - 4\hat{j} + 12\hat{k}) - 9 = 0$.

46. Show that the points $\hat{i} - \hat{j} + 3\hat{k}$ and $3\hat{i} + 3\hat{j} + 3\hat{k}$ are equidistant from the plane $\vec{r}.(5\hat{i} + 4\hat{j} - 7\hat{k}) + 9 = 0$.

47. Show that the point (1, 1, 1) and (–3, 0, 1) are equidistant from the plane $3x + 4y - 12z + 13 = 0$.

48. Find the equations of the planes parallel to the plane $x - 2y + 2z - 3 = 0$ and which are at a unit distance from the point (1, 1, 1).

49. Fidn the equation of the plane mid-parallel to the planes

$$2x - 2y + z + 3 = 0 \text{ and } 2x - 2y + z + 9 = 0.$$

50. Find the distance between the planes $\vec{r}.(\hat{i} + 2\hat{j} + 3\hat{k}) + 7 = 0$ and $\vec{r}.(2\hat{i} + 4\hat{j} + 6\hat{k}) + 7 = 0$.

51. If the line $\vec{r}.(\hat{i} - 2\hat{j} + \hat{k})\lambda(2\hat{i} + \hat{j} + 2\hat{k})$ is parallel to the plane $\vec{r}.(3\hat{i} - 2\hat{j} + m\hat{k}) = 14$, fund the value of m.

52. Find the vector equation of the line passing thorugh the point with position vector $2\hat{i} - 3\hat{j} - 5\hat{k}$ and perpendicular to the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 5\hat{k}) + 2 = 0$.

53. Find the equation of the plane passing through the line of intersection of the plane $2x + y - z = 3$, $5x - 3y + 4z + 9 = 0$ and parallel to the line $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

54. Find the equation of the plane passing through the point A (1, 2, 1) and perndicular to the line joining the points P(1, 4, 2) and Q(2, 3, 5). Also, Find the distance of this plane from the line $\frac{x+3}{2}=\frac{y-5}{-1}=\frac{z-7}{-1}$

55. Find the angle between the line $\vec{r}.(2\hat{i}+3\hat{j}+9\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k})$ and the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})$ = 5.

56. Find the angle between the line $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z+1}{1}$ and the plane $2x + y - z = 4$.

57. The line $\vec{r}=\hat{i}+\lambda(2\hat{i}-m\hat{j}-3\hat{k})$ is parallel to the plane $\vec{r}.(m\hat{i}+3\hat{j}+\hat{k})=4$. find m.

58. Show that the lire whose vector equation is $\vec{r}=2\hat{i}+2\hat{j}+7\hat{k}+\lambda(\hat{i}+3\hat{j}+4\hat{k})$ is parallel to the plane whose vector equation is $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=7$. Also, find the distance between them.

59. Find the equation of the line passing thorugh (1, 2, 3) and parallel to the planes $x - y + 2z = 5$ and $3x + y + z = 6$.

60. Find the equation of the plane passing through the intersection of the planes $x - 3y + z = 1$ and $2x + y + z = 8$ and parallel to the line with direction ratios proportional to 1, 2, 1. Find also the perpendicular distance fo (1, 1, 1) from this plane.

61. State when the line $\vec{r}=\vec{a}+\lambda\vec{b}$ is parallel to the plane $\vec{r}.\vec{n}=d$. Show that the line $\vec{r}=\hat{i}+\hat{j}+\lambda(3\hat{i}-\hat{j}+2\hat{k})$ is parallel to the planer $\vec{r}=(2\hat{i}+\hat{k})=3$. Also, find the distance between the line and the plane.

62. Find the equation of the plane thorugh the intersection of the planes $3x - 4y + 5z = 10$ and $2x + 2y - 3z = 4$ and parallel to the line $x = 2y = 3z$.

63. Find the equation of the plane passing through the points (3, 4, 1) and (0, 1, 0) and parallel th the line $\frac{x+3}{2}=\frac{y-3}{7}=\frac{z-2}{5}$

64. Find the coordiates of the point where the line thorugh the points A (3, 4, 1) and B (5, 1, 6) crosses the XY-plane.

65. Show that the lines

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \text{ and } \frac{x-b+c}{\alpha-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma} \text{ are coplanar.}$$

66. Find the equation of the plane passing thorugh the point (0, 7, –7) and containing the line $\frac{z}{1}=\frac{y-7}{}=\frac{x+1}{-3}$.

67. Show that the plane whoxe vector equation is $\vec{r}.(\hat{i}+2\hat{j}-\hat{k})=3$ contains the line whose vector equation is $\vec{r}=(\hat{i}+\hat{j})+\lambda(2\hat{i}+\hat{j}+4\hat{k})$.

68. Find the vector and cartesian equation of the plane containing the two line

$$\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda(\hat{i} + 2\hat{j} + 5\hat{k})$$

and, $\vec{r} = 3\hat{i} + 3\hat{j} - 2\hat{k} + \mu(3\hat{i} + 2\hat{j} + 5\hat{k})$

69. If $4x + 4y - \lambda z = 0$ is the equation of the plane thorugh the origin that contawins the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{4}$, find the value of λ.

70. Show that the line $\vec{r} = (2\hat{i} - 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 5\hat{k})$ are coplanar. Also, find the equation of the plane containing them.

71. Show that the lines $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ are coplanar. Also find the equation of the plane containing them.

72. Find he equation of the plane $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ and the point (0, 7, –7) and show that the line $\frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$ aslo lines in the same plane.

73. Find the distance of the point (–1, –5, –10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} - 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}\cdot(\hat{i} - \hat{j} + \hat{k}) = 5$.

74. Find the leanght and the foot of the perpendicular from the point (7, 14, 5) to the plane $2x + 4y - z = 2$.

75. Find the image of the point having position vector $\hat{i} + 3\hat{j} + 4\hat{k}$ in the plane $\vec{r}\cdot(2\hat{i} - \hat{j} + \hat{k}) + 3 = 0$.

76. Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured along a line parallel to $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

77. Find the lenght and the foot of the perpendicular from the point (1, 1, 2) to the plane $\vec{r}\cdot(\hat{i} - 2\hat{j} + 4\hat{k}) + 5 = 0$.

78. Find the value of λ such that the line $\frac{x-2}{6} = \frac{y-1}{\lambda} = \frac{z+5}{-4}$ is perpendicular to the plane $3x - y - 2z = 7$.

79. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point P(5, 2, 1) from the plane $2x - y + z + 1 = 0$. Find also the image of the point in the plane.

80. Write the distance between the parallel planes $2x - y + 3z = 4$ and $2x - y + 3z = 18$.

81. Write the value of k for which the line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{k}$ is perpendicular to the normal to the plane $\vec{r}\cdot(2\hat{i} + 3\hat{j} + 4\hat{k}) = 4$.

82. Write the angle between the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z+3}{-2}$ and the plane $x + y + 4 = 0$.

EXERCISE - 2

1. If al line makes angles of 90^0, 60^0 and 30^0 with the postive direction of x, y, and z=axis respectively, fund its direction cosines.
2. Find the direction cosines of the line passing through the rwo points (–2, 4, –5) and (1, 2, 3).
3. Find the vector and cartesian equations of the line through the point (5, 2, –4) and which is paralled to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.
4. Show that the three lines with direction cosines $\frac{13}{13},\frac{-3}{13},\frac{-4}{13}$; $\frac{4}{13},\frac{12}{13},\frac{3}{13}$; $\frac{3}{13},\frac{-4}{13},\frac{12}{13}$ and mutually perpendictar.
5. Find the shortest distance between the lines

   (i) $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\gamma(\hat{i}-\hat{j}-\hat{k})$ and, $\vec{r}=2\hat{i}-\hat{j}-\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$

   (ii) $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$ and $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$

   (iii) $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\gamma(\hat{i}-3\hat{j}+2\hat{k})$ and, $\vec{r}=4\hat{i}+5\hat{j}+6\hat{k}+\mu(2\hat{i}+3\hat{j}+\hat{k})$

   (v) $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\gamma(\hat{i}-2\hat{j}+2\hat{k})$ and, $\vec{r}=4\hat{i}-6\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$
6. Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points (3, 5, –1) and (4, 3, –1).
7. Find the vector equation of the line passing through the point (1, 2, –4) and perpendicular to the lines :

   $$\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7} \text{ and } \frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$
8. Find the vectro equation of the plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vectro from the origin is $2\hat{i}-3\hat{j}+4\hat{k}$. Aslo, find its cartesian from.
9. Find the directions cosines of the unit vector perpendicular to the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})+1=0$ passing through the origin.
10. Show that lines

    $$\frac{x+4}{-3}=\frac{y-1}{1}=\frac{z-5}{5} \text{ and } \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$$
11. Find the angle between the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$.
12. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

13. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$.
14. Find the equation of the plane passing through the points (1, 1, –1), (6, 4, 5) and (–4, –2, 3).
15. Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.
16. Find the equation of the plane passing thorugh (a, b, c) and parallel to the plane $\vec{r}.(\hat{i} + \hat{j} + \hat{k}) = 2$.
17. Find the equation of the plane passing through the point (–1, 3, 2) and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.